T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

# MODEL SOLUTIONS TO IIT JEE 2008
# PAPER 1
# CODE - 0

## PART I

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| **B** | **C** | **A** | **B** | **C** | **C** |

| 7 | 8 | 9 | 10 |
|---|---|---|---|
| **B, D** | **B, C** | **A, D** | **A, B, C, D** |

| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|
| **B** | **D** | **A** | **B** | **D** | **A** | **D** |

| 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|
| **B** | **A** | **D** | **B** | **C** | **D** |

## Section I

1. $y^2 = 4x$ — (1)
$x^2 + y^2 - 6x + 1 = 0$ — (2)
Put (1) in (2), $x^2 - 2x + 1 = 0 \Rightarrow x = 1$
Now (1) gives $y = \pm 2$
$C_1$ and $C_2$ intersect at (1, 2) and (1, –2) or touch each other at these points.
Now for (1), $y' = \frac{2}{y}$ — (3) and for (2),
$y' = \frac{3 - x}{y}$ — (4)
At 1, 2) value of y' given by (3) and (4) are equal each equal to 1
Similarly at (1, –2) value of y' = –1 (for (3) and (4))
$\therefore \Rightarrow$ (b)

2. $\cot^{-1}x = \sin^{-1}\frac{1}{\sqrt{1+x^2}} = \cos^{-1}\frac{x}{\sqrt{1+x^2}}$
$\therefore$ General expression = $x\sqrt{1+x^2}$
$\therefore \Rightarrow$ (c)

3. Let $a_1 i, b_1 i + b_2 j$ , $c_1 i + c_2 j + c_3 k$ be the vectors
Let $a_1 = 1 \Rightarrow \bar{a} \cdot \bar{b} = \frac{1}{2}$
$\therefore b_1 = \frac{1}{2}; |\bar{b}| = 1 \quad \Rightarrow b_2 = \frac{\sqrt{3}}{2}$
$\bar{a}\cdot\bar{c} = \frac{1}{2}; \quad \therefore c_1 = \frac{1}{2};$
$\bar{b}\cdot\bar{c} = \frac{1}{2} \Rightarrow b_1c_1 + b_2c_2 = \frac{1}{2}$
$\frac{1}{4} + \frac{\sqrt{3}}{2}c_2 = \frac{1}{2}$
$c_2 = \frac{1}{4} \times \frac{2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}$
$|\bar{c}| = 1 \Rightarrow c_3 = \frac{\sqrt{2}}{\sqrt{3}}$
$\therefore \bar{a} = \hat{i}$ ; $\bar{b} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}; \bar{c} = \frac{1}{2}\hat{i} + \frac{1}{2\sqrt{3}}\hat{j} + \frac{\sqrt{2}}{\sqrt{3}}\hat{k}$

$$\therefore \left[\bar{a}, \bar{b}, \bar{c}\right] = \begin{vmatrix} 1 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 1/2 & 1/2\sqrt{3} & \sqrt{2}/\sqrt{3} \end{vmatrix}$$

$$= 1 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ (a)

4. Given equation can be written as $(ax^2 + by^2 + c)(x - 3y)(x - 2y) = 0$ — (1)
If c = 0, a & b of the same sign
Now $ax^2 + by^2 = 0$ if and only if x = 0 = y
$\therefore$ (a) is false
when a = b, c is of sign opposite to that of (a)
then (1) represents (2) straight lines and a circle
$\therefore \Rightarrow$ (b)

5. $h(x) = |x - 1| = \begin{cases} 1 - x, & \text{if } x \le 1 \\ x - 1 & \text{if } x > 1 \end{cases}$

$h'(x) = \begin{cases} -1, & \text{if } x < 1 \\ 1 & \text{if } x > 1 \end{cases}$

$\therefore$ p = LHD of h(x) at x = 1 = –1
Let h = x – 1

$$\therefore -1 = \lim_{x \to 1^+} g(x) = \lim_{h \to 0} \frac{h^n}{m(\log \cosh)} \left(\frac{0}{0} \text{form}\right)$$

$$= \lim_{h \to 0} \frac{-n}{m} \frac{h}{\sinh} \quad h^{n-2} \cosh$$

$$-1 = \lim_{h \to 0} \frac{-nh^{n-2}}{m\left(\frac{\tan h}{h}\right)}$$

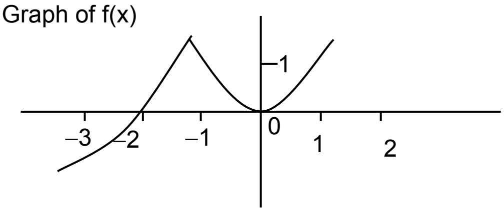
Given n > 0 and $\lim_{x \to 1^+} g(x) = -1$

$\therefore$ n = 2

$\Rightarrow -1 = \frac{-2}{m}$

$\therefore$ m = 2
$\Rightarrow$ m = n = 2
$\Rightarrow$ (c)

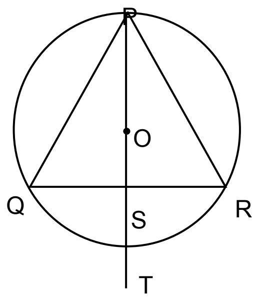
6. Graph of f(x)




2 local extreme
$\therefore \Rightarrow$ (c)

## Section II

7.




Let Q(0, 0) R(a, 0), $P\left(\frac{a}{2}, \frac{\sqrt{3}}{2}a\right)$ so that

$S = \left(\frac{a}{2}, 0\right)$ PS $\times$ ST = QS $\times$ SR

$$\Rightarrow ST = \frac{QS \times SR}{PS} = \frac{\frac{a}{2} \times \frac{a}{2}}{\frac{\sqrt{3}}{2}a} = \frac{a}{2\sqrt{3}}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} = \frac{2}{\sqrt{3}a} + \frac{2\sqrt{3}}{a} = \frac{8}{\sqrt{3}a}$$

$$= \frac{8\sqrt{3}}{3a} \quad \text{— (1)}$$

$$\frac{2}{\sqrt{QS \times SR}} = \frac{2}{\sqrt{\frac{a}{2} \cdot \frac{a}{2}}} = \frac{4}{a} \quad \text{— (2)}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$$

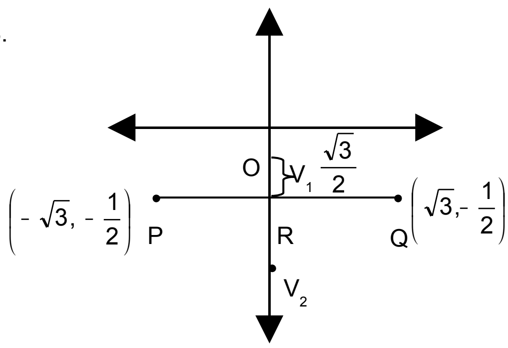
$\Rightarrow$ (b)

Also $\frac{1}{PS} + \frac{1}{ST} = \frac{8\sqrt{3}}{3a}$

$\frac{4}{QR} = \frac{4}{a}$, so that $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

$\Rightarrow$ (d)

8.




$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \rightarrow e^2 = \frac{4 - 1}{4} = \frac{3}{4} \Rightarrow ae = \sqrt{3}$$

& x = $\sqrt{3}$ $\Rightarrow y^2 = \frac{1}{4} \Rightarrow y = \pm\frac{1}{2}$

$\Rightarrow P\left(\sqrt{3}, -\frac{1}{2}\right), Q\left(-\sqrt{3}, -\frac{1}{2}\right)$

PQ = 4a = $2\sqrt{3} \Rightarrow a = \frac{\sqrt{3}}{2}$

R(0, $-\frac{1}{2}$) $\Rightarrow V_1\left(0, -\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$ and $V_2$

$\left(0, -\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

$\Rightarrow$ Parabola (1) $\Rightarrow (x - 0)^2 = -2\sqrt{3}\left(y + \frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

Or $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

$\Rightarrow$ (c)

Parabola (2) $\Rightarrow (x - 0)^2 = 2\sqrt{3}\left(y + \frac{1}{2} + \frac{\sqrt{3}}{2}\right)$

$\Rightarrow x^2 - 2\sqrt{3}y = 3 + \sqrt{3} \rightarrow$ (b)

$\Rightarrow$ (b)

9. $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{k=1}^{n} \frac{n^2}{n^2 + kn + k^2}$

a = 0, b = 1

$h = \frac{b - a}{n} = \frac{1}{n}$

$= \lim\limits_{n\to\infty} \frac{1 - 0}{n}\sum\limits_{k=1}^{n} \frac{1}{1 + \left(\left(\frac{k}{n}\right)^2 + \frac{k}{n}\right)}$

$= \frac{1}{n}\sum\limits_{k=1}^{n} \frac{1}{\frac{3}{4} + \left(\frac{k}{n} + \frac{1}{2}\right)^2}$

$= \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$

$= \frac{2}{\sqrt{3}} \tan^{-1} \left.\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]_0^1$

$= \frac{2}{\sqrt{3}}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{\pi}{3\sqrt{3}}$

Similarly $\lim\limits_{n\to\infty} T_n = \frac{\pi}{3\sqrt{3}}$

Hence $S_n < \frac{\pi}{3\sqrt{3}} < T_n$ or $T_n < \frac{\pi}{3\sqrt{3}} < S_n$

But $S_1 = \frac{1}{3}$ $\quad T_1 = 1$

$\Rightarrow S_1 < \frac{\pi}{3\sqrt{3}} < T_1$

$\Rightarrow S_n < \frac{\pi}{3\sqrt{3}} < T_n$

$\therefore \Rightarrow$ (a) & (d)

10. Let f(x) = f(1 – x) and $f'\left(\frac{1}{4}\right) = 0$

$f'(x) = -f'(1 - x)$

$f'\left(\frac{1}{2}\right) = -f'\left(\frac{1}{2}\right) \Rightarrow f'\left(\frac{1}{2}\right) = 0$

$\Rightarrow$ (b)

Given $f'\left(\frac{1}{4}\right) = 0$ and by (b) $f'\left(\frac{1}{2}\right) = 0$

$\Rightarrow$ f'(x) vanishes at 2 points $\frac{1}{4}$ and $\frac{1}{2}$.

f(x) = f(1 –x).

$\Rightarrow$ f(0) = f(1); f(–1) = f(2) etc

$\Rightarrow f\left(x - \frac{1}{2}\right) = f\left(x + \frac{1}{2}\right)$

$\therefore$f(x) is symmetric about x = $\frac{1}{2}$ — (2)

From (1) and (2), f'(x) vanishes at 4 points in [0, 1]

$\Rightarrow$f''(x) vanishes at 2 points in [0, 1]

$\Rightarrow$ a

$\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x \, dx$

Let $I(x) = f\left(x + \frac{1}{2}\right) \sin x$

$I(-x) = f\left(-x + \frac{1}{2}\right) \sin(-x)$

$= -f\left(\frac{1}{2} - x\right) . \sin x = -f\left(x + \frac{1}{2}\right) \sin x$

$= -I(x)$

$\therefore$ I(x) is odd function

$\therefore \int_{-1/2}^{1/2} I(x)\,dx = 0$

$\int_{1/2}^{1} f(1 - t)e^{\sin\pi t}dt$ $\quad$ $1 - t = x$, $t = 1 - x$, $dt = -dx$

$\int_{1/2}^{0} f(x).e^{\sin\pi t}$

| | |
|---|---|
| $t = \frac{1}{2}$ | $x = \frac{1}{2}$ |
| $t = 1$ | $x = 0$ |

$$\int_{1/2}^{0} f(x).e^{\sin\pi(1-x)} - dx$$

$$= -\int_{1/2}^{0} f(x).e^{\sin(\pi - \pi h)}dx$$

$$= \int_{0}^{1/2} f(x)e^{\sin\pi x}dx$$

$\Rightarrow$ (b) & (d)

## Section III

11. Limit = $\lim_{x\to 0}\left(\frac{g(x)\cos x - g(0)}{\sin x}\right)\left(\frac{0}{0}\right)$

$= \lim_{x\to 0}\left(\frac{g'(x)\cos x - g(x)\sin x}{\cos x}\right)$

$= \lim_{x\to 0}\left(\frac{g'(x)\cos x - f(x)}{\cos x}\right)$

= 0

Also f”(0) = 0

$\therefore$ Statement 1 is true

$\therefore$ Statement 2 is also true.

But not using to prove statement (1)

$\therefore \Rightarrow$ (b)

12. Directions ratios of the line ($L_3$) represented by $P_1$ and $P_2$ are 0, 2, 2.
Likewise D.r.s of $L_1$ and $L_2$ are 0, –4, –4 and 0, –2, –2 respectively
This means, $P_1$, $P_2$, $P_3$ form a triangular prism.
$\therefore$Statement 1 is false.
$\therefore \Rightarrow$ (d)

13. $\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

= 1 (4 – 6) + 2 (–4 + 2) + 3 (3 – 1)

= –2 – 4 + 6 = 0

$\Delta_1 = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \begin{vmatrix} 0 & -5 & 7 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

= 5 (4k + 2) + 7 (–3k – 1)

= 20k + 10 – 21k – 7

= –k + 3

$k \neq 3 \Rightarrow$ No solution

Likewise $\Delta_3$ = – k + 3

$\Rightarrow$ No solution for $k \neq 3$

$\Rightarrow$ (a)

14. Determinant of the coefficients = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$
There will be 16 determinants in total, out of which 6 are non zero given below:
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\Rightarrow$ (b)

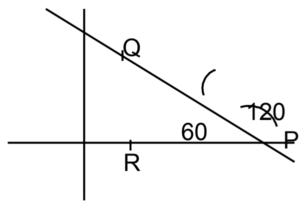
15. Equation of PQ is $\sqrt{3}x + y = 6$

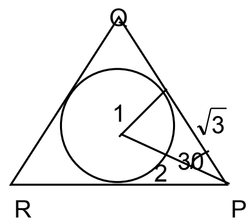
$\Rightarrow$ slope two = $-\sqrt{3}$

$\therefore \theta = 120$

$\therefore$ PR in line parallel to x = axis

$\therefore$ Equation PR in y = 0




Using circumcircle, radius from the figure

PD = $\sqrt{3}$ = r $\left(\tan 30 = \frac{1}{r} = \frac{1}{\sqrt{3}}\right)$

$\therefore$ any point at a distance r on PQ uniform D $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$

i.e. $\left(\frac{3\sqrt{3}}{2} \pm \sqrt{3}\cos 120 \; \frac{3}{2} \pm \sqrt{3}\sin 120\right)$

$\therefore P\left(\frac{3\sqrt{3}}{2} - \sqrt{3}.\frac{-1}{2} \quad \frac{3}{2} - \sqrt{3}.\frac{\sqrt{3}}{2}\right)$

$= P\left(2\sqrt{3},\ 0\right)$

$Q\left(\frac{3\sqrt{3}}{2} + \sqrt{3}\,\frac{-1}{2} \; \frac{3}{2} + \frac{\sqrt{3}.\sqrt{3}}{2}\right)$

$= Q\left(\sqrt{3}, 3\right)$

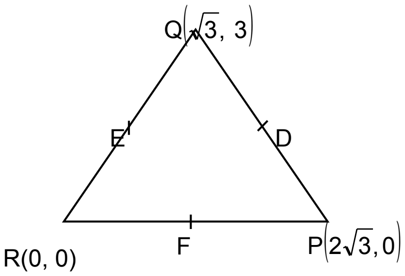
since PQR is equilateral PQ = $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$

$\therefore$ PQ = $2\sqrt{3}$ = RP

since equation of RP in y = 0 & origin in left side of PQ, RP = $2\sqrt{3}$ & $P\left(2\sqrt{3},\ 6\right)$

$\Rightarrow$ R in origin (0, 0)

$\therefore$




$\Rightarrow E \Rightarrow \left(\dfrac{\sqrt{3}+0}{2},\ \dfrac{3+0}{2}\right) \Rightarrow \left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right)$

$F \Rightarrow \left(\dfrac{0+2\sqrt{3}}{2}, \dfrac{0+0}{2}\right) = \left(\sqrt{3}, 0\right)$

Centre of circle c = centroid of $\Delta DEF$

$\Rightarrow \left(\dfrac{x_1+x_2+x_3}{3},\ \dfrac{y_1+y_2+y_3}{3}\right)$

$\Rightarrow \left(\sqrt{3},\ 1\right)$

$\therefore$ Equation of circle is $\left(x-\sqrt{3}\right)^2 + (y-1)^2 = 1$

$\Rightarrow$ (d)

16. From the solution to 15 $\Rightarrow$ (a)

17. Equation RP is y = 0

Equation RQ is $\dfrac{y-0}{3-0} = \dfrac{x-0}{\sqrt{3}-0}$

$y = \sqrt{3}x$

$\Rightarrow$ (d)

## Section IV

18. $f(x)^3 - 3f(x) + x = 0$

$3f(x)^2 f'(x) - 3f'(x) + 1 = 0$

$3(f(-10\sqrt{2}))^2 f'\left(-10\sqrt{2}\right) - 3f'\left(-10\sqrt{2}\right) + 1 = 0$

$3\left(2\sqrt{2}\right)^2 f'\left(-10\sqrt{2}\right) - 3f' + 1 = 0$

$21 f'\left(-10\sqrt{2}\right) + 1 = 0$

$f'(-10\sqrt{2}) = -\dfrac{1}{21}$

$6f(x)\, f'(x)^2 + 3.f(x)^2 f''(x) - 3f''(x) = 0$

$6\left(2\sqrt{2}\right)\left(\dfrac{1}{21}\right)^2 +$

$3\left(2\sqrt{2}\right)^2 f''\left(-10\sqrt{2}\right) - 3.f''\left(-10\sqrt{2}\right) = 0$

$21f'' = -\dfrac{4\sqrt{2}}{147}$

$f'' = \dfrac{-4\sqrt{2}}{21^2 \times 7} = \dfrac{-4\sqrt{2}}{7^3 \times 3^2}$

19. $y^3 - 3y + x = 0 \Rightarrow y' = \dfrac{-1}{3\left(y^2-1\right)}$

$= \dfrac{-1}{3\left(f(x)^2-1\right)}$

Now, $\displaystyle\int_a^b x.f'(x)\,dx$

$= \left(xf(x)\right)_a^b - \displaystyle\int_a^b 1.f(x)\,dx$

$\therefore \displaystyle\int_a^b f(x)\,dx = bf(b) - af(a) - \int_a^b x.\dfrac{-1}{3\left(f(x)^2-1\right)}$

= b

$= \displaystyle\int_a^b \dfrac{x}{3\left[(f(x))^2-1\right]}\,dx + bf(b) - a\,f(a)$

$\Rightarrow$ (a)

20. g(x) is defined in (–2, 2) and g(0) = 0

$\Rightarrow$ g(x) is an odd function

$\therefore$ g'(x) is an even function

$\therefore \displaystyle\int_{-1}^{1} g'(x)\,dx = 2\int_0^1 g'(x)\,dx$

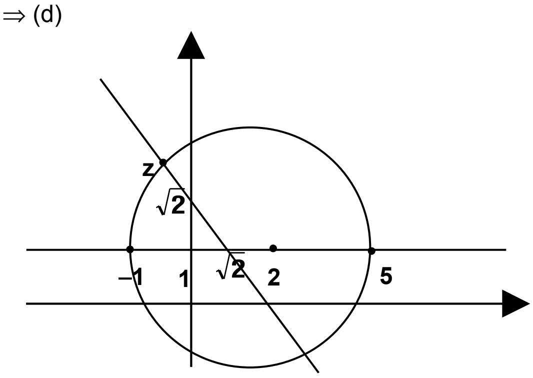
$= 2\left[g(x)\right]_0^1$

= 2(g(1) – g(0) (∵g(0) = 0)

= 2g(1)

$\Rightarrow$ (d)

21.




A represents the region above the line y = 1

B represents the circle with centre at (2, 1) and radius 3.

C represents the straight line $x + y = \sqrt{2}$

Re(z (1 – i) = Re [(x + iy) (1 – i)] = $x + y = \sqrt{2}$ (given)

From the figure it is clear that $A \cap B \cap C$ contains only one element
$\therefore \Rightarrow$ (b)

22. Observe that (–1, 1) and (5, 1) are points on the extremities of a diameter and z is a point on the semicircle

$\therefore$ (–1, 1), z, and (5, 1) form a right angled triangle
$\therefore |z + 1 - i|^2 + |z - 5 - i|^2 = 6^2 = 36$
$\therefore \Rightarrow$ (c)

23. $\| Z | - | W \| \le | Z - W|$
$\le |Z - (2 + i) - (W - (2 + i)|$
$\le | Z - (2 + i) | - | W - (2 + i) |$
$< | Z - (2 + i) | + | W - (2 + i) |$
$| |Z| - |W| | \le 6$
$-6 \le | Z | - | W | < 6$
$-3 \le |Z | - | W | + 3 \le 9$
$\Rightarrow$ (d)

# PART II

| 24 | 25 | 26 | 27 | 28 | 29 |
|---|---|---|---|---|---|
| **B** | **C** | **B** | **A** | **C** | **C** |

| 30 | 31 | 32 | 33 |
|---|---|---|---|
| **A, D** | **B, D** | **A, C, D** | **A, B** |

| 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|---|---|
| **D** | **A** | **D** | **B** | **D** | **B** | **B** |

| 41 | 42 | 43 | 44 | 45 | 46 |
|---|---|---|---|---|---|
| **C** | **C** | **A** | **B** | **B** | **C** |

## Section I

24. $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$

$E_1 = \frac{0.1}{64} + 2 \times \frac{0.1}{128}$

$E_2 = \frac{0.1}{64} + 2 \times \frac{0.1}{64}$

$E_3 = \frac{0.1}{20} + 2 \times \frac{0.1}{36}$

25. $R_1$ = Balanced Wheatstone = 1 Ω
$R_2$ = Parallel combination = 0.5 Ω
$R_3$ = series parallel combination = 2 Ω
$P_2 > P_1 > P_3$

26. Cut off wavelength depends only on the accelerating voltage

27. $r_1 = r_2$ for minimum deviation
$= \frac{A}{2} = 30$ for all colours

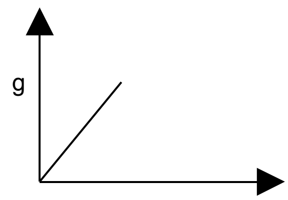
28. $PT^2$ = constant

$\frac{dP}{P} + \frac{2dT}{T} = 0$ ; PV = nRT

$\Rightarrow \frac{dP}{P} + \frac{dV}{V} = \frac{dT}{T}$

$\Rightarrow \frac{dV}{V} = \frac{3dT}{T}$

29.




$\frac{mv^2}{r} = mg$

$v = \sqrt{gr}$

$r < R - v \propto r$

$v > R - v \ \frac{1}{\sqrt{r}}$

## Section II

30. (A) $C_1 \neq 0$ not allowed
(D) $\hat{j}$ comp. = 2 $b_1$ and not allowed

31. Total binding energy of the products should be large.

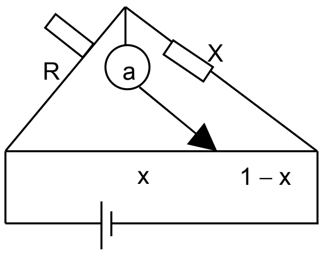
32. Circular path $r = \frac{mv}{qB}$

$\omega = \frac{qB}{m}$

33. If d = λ, path difference will be equal to λ only at central maxima. So A is correct. Similarly B is also true.

## Section III

34.




$\frac{X}{R} = \frac{1-x}{x}$

when temperature increases X increases; so that R has to be increased to keep x same

35. In the space station frame of reference, the astronaut experiences no force. Centrifugal force cancels the gravitational force.

36. $T = \frac{\sqrt{2gh\left(1+\frac{K^2}{R^2}\right)}}{\sin\theta}$

$\frac{K^2}{R^2}$ is greater for hollow cylinder.

37. Bernoulli equation is also required to explain. Steady flow equation only gives Av is constant. The decrease/increase in v has to be explained.

## Section IV

38. Pressure force is the buoyant force

39. $PV^{\gamma}$ = constant ⇒ $T \propto P^{1-\frac{1}{\gamma}}$
$P = P_0 + \rho - \gamma)$

40. $F_b = \rho \frac{nRT}{P} g$

41. H atom from n = 2 state to ground n = 1
⇒ $H_e^+$ from n = 2 to n = 4

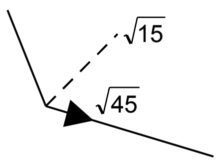
42. n = 4 → n = 3
$E = 4 \times 13.6\left(\frac{1}{9} - \frac{1}{16}\right)$

43. $E \propto Z^2$

44. $H = \sqrt{3}\tan 60 = 3$
$v = \sqrt{2gH} = \sqrt{60}$
$v_B = v\cos 30 = \sqrt{45}$

45. $u = 3\sqrt{3}\tan 30 = 3$
$v^2 = u^2 + 2gh \Rightarrow v = \sqrt{105}$

46.




$\sqrt{15}\sin 60 - \sqrt{45}\sin 30 = 0$

# PART III

| 47 | 48 | 49 | 50 | 51 | 52 |
|---|---|---|---|---|---|
| **B** | **A** | **B** | **B** | **A** | **D** |

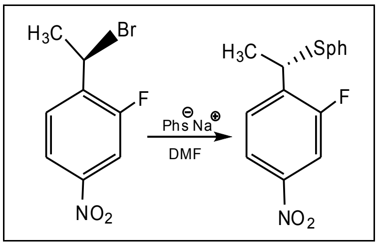
| 53 | 54 | 55 | 56 |
|---|---|---|---|
| **A, D** | **B, C, D** | **A, B** | **A, C, D** |

| 57 | 58 | 59 | 60 | 61 | 62 | 63 |
|---|---|---|---|---|---|---|
| **C** | **C** | **A** | **D** | **D** | **A** | **C** |

| 64 | 65 | 66 | 67 | 68 | 69 |
|---|---|---|---|---|---|
| **C** | **C** | **B** | **D** | **B** | **B** |

## Section I

47. Hyperconjugation is the, overlap of the σ with the p orbitals.

48. 


The presence of nitro group in the meta position does not influence the replacement of the nuclear fluorine. This is an $S_N2$ reaction occurring in a polar aproatic solvent dimethyl formide $HCON(CH_3)_2$. Hence the reaction proceds by inversion of configuration.

49. $Na_2S_2O_3 + 4Cl_2 + 5H_2O \rightarrow$
$2NaHSO_4 + 8HCl$

50. Silver dissolves in aq.NaCN in presence of air (oxygen).
$4Ag + 8NaCN + 2H_2O + O_2 \rightarrow$
$4Na[Ag(CN)_2] + 4NaOH$

51. $K_1 = \frac{0.693}{T_{1/2}}$

$K_0 = \frac{A_o}{2T_{1/2}} = \frac{1.386}{2 \times 20} = \frac{0.693}{40}$

$\frac{K_1}{K_0} = 0.5 \text{ mol}^{-1} \text{ dm}^3$

52. 2.5 mL $\frac{2}{5}$ M base is neutralized by $\frac{15}{2}$ mL $\frac{2}{15}$ M acid

Salt concentration = $\frac{1}{10}$

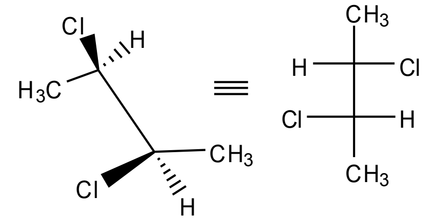
C = 0.1

$\frac{K_w}{K_b} = \frac{Ch^2}{1-h}$

On solving, h = 0.27

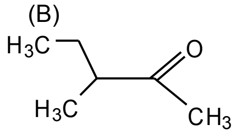
$[H^+] = Ch = 2.7 \times 10^{-2}$ M

## Section II

53. 

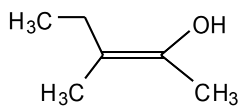

The compound is optically active.
The compound possess two fold axis of symmetry.

54. The correct statements concerning E, F & G are (B)



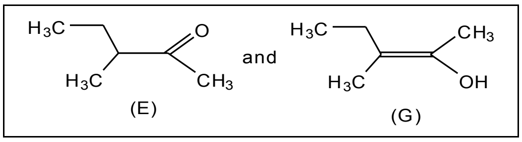

(E)

and



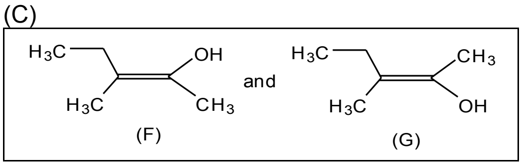

(F)

are tautomers,




(E) and (G) are tautomers

(C)




(D)

F and G are diasteromers because geometic isomers have diastereomeric relationship.

55. H = $NH_4NO_3$ or $NH_4NO_2$

With NaOH they produce $NH_3$

$NH_4NO_3 + NaOH \rightarrow NaNO_3 + NH_3 + H_2O$

$NH_4NO_2 + NaOH \rightarrow NaNO_2 + NH_3 + H_2O$

After evolution of $NH_3$ ceases Zn dust liberates $NH_3$ by reduction of $NO_3^-$ or $NO_2^-$

$$4Zn + 7OH^- + 6H_2O + NO_3^- \rightarrow NH_3\uparrow + 4Zn(OH)_4^{2-}$$

$$3Zn + 5OH^- + 5H_2O + NO_2^- \rightarrow NH_3 + 3Zn(OH)_4^{2-}$$

56. In the limit of large molar volume $\frac{'a'}{V^2}$ and 'b' are negligibly small, hence, PV = RT. 'a' and 'b' are characteristics of a gas but they are temperature independent. Real pressure is less than the ideal pressure.

## Section III

57. Statement (1)

Bromobenzene on reaction with $Br_2$ / Fe gives 1,4-dibromo benzene as the major product. This is nuclear bromination of bromobenzene. The statement (1) is correct.

Statement (2)

In bromobenzene, the inductive effect of the bromo group is dominant than the mesomeric effect in directing the incoming electrophile.

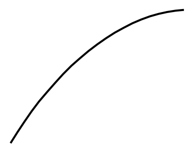
In bromobenzene, there are two competing effects, viz inductive effect which is deactivating while the mesomeric effect which is activating. Between the two resonance or mesomeric effect dominates. Hence bromobenzene is deactivating, but ortho, para directing.

Hence statement (2) is not correct.

58. Statement –1 is true, statement –2 is false.

Higher oxidation states for group 14 elements are less stable for the heavier members of the group due to inert "pair effect".

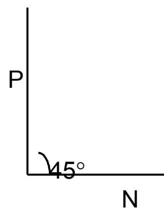
59.



Statement (1) is true, statement (2) is true and is a correct explanation for statement (1)

60. The Gibbs free energy change for a reaction is zero at equilibrium i.e., $\Delta G = 0$ and not $\Delta G° = 0$.

For a spontaneous chemical reaction $\Delta G_{(T,\ P)}$ is negative




## Section IV

$$H_3C-CH_2-C\equiv C-CH_2-CHO \xrightarrow{NaBH_4}$$

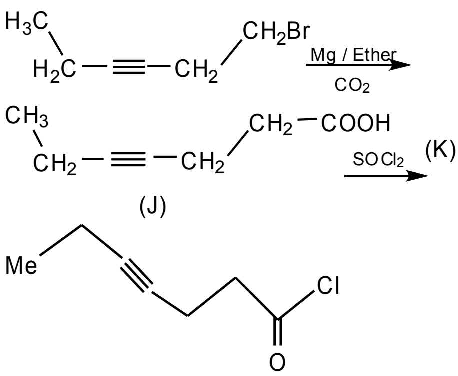
Hex-3-ynal

61. $H_3C-CH_2-C\equiv C-CH_2-CH_2OH \xrightarrow{PBr_3}$

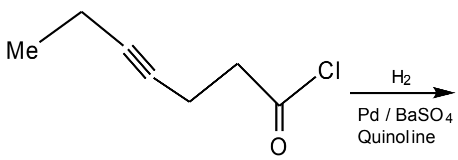
$$CH_3CH_2-C\equiv C-CH_2-CH_2Br$$

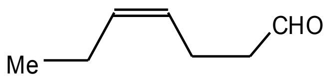
(I)

62.




63.





(L)

The triple bond is reduced to the cis double bond. The acid chloride is reduced to the aldehyde.

64. Nitrates are less abundant than phosphate because the former are soluble in water and the latter are insoluble.

65. $NH_3$ is a better electron donor because the lone pair of electrons occupies $sp^3$ orbital which is more directional.

66. disproportionation reaction

$P_4^0 + 3NaOH + 3H_2O \rightarrow 3NaH_2\overset{+1}{P}O_2 + \overset{-3}{P}H_3$

67. Molality of the solution = $\frac{0.1 \times 1000}{0.9 \times 46} = 2.41$

depression of F.P = $2.41 \times k_f$
= 4.83K

F.P of solution = 155.7K – 4.83K = 150.9K

68. Molefraction of ethanol = 0.9

V.P of solution $p_s = 40 \times 0.9 = 36$ mm of Hg

69. Molality of the solution = $\frac{0.1 \times 1000}{0.9 \times 18} = 6.17$ m

Elevation of b.p = $k_b m$
= $0.52 \times 6.17$
= 3.2K

B.P of solution = 373 + 3.2 = 376.2K